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E. J. STONE, M.A., F.R.S., President, in the Chair.

Lieut. William Heron Coombs, R.N., Indian Marine Survey;
Thomas Kilner Mellor, 3 Belmont Street, Huddersfield;
John Pender, M.P., 18 Arlington Street, W.; and
William John Vernon Vandenberg, F.M.S., Hornsey, Middlesex;

were balloted for and duly elected Fellows of the Society.

Note on Delaunay's Expression for the Moon's Parallax.

By Professor J. C. Adams, M.A., F.R.S.

The process employed in Delaunay's Theory of the Moon consists in making a great number of successive changes from one system of elements to another, these changes being so conducted that the equations which give the variations of the elements always retain their canonical form, until at length all the sensible periodic terms in the disturbing function are got rid of, and the elements are thus reduced to three constants and three angles which vary in proportion to the time.

After each such change of elements, the expressions for the three coordinates of the Moon, which are supposed to be known in terms of the old system of elements, must be transformed so as to be expressed in terms of the new.

These transformations being made independently, we may, if we choose, find some of the coordinates with a greater degree of precision than others.

Delaunay has, as is well known, followed the example of Plana in developing his coefficients in series of ascending powers of the small quantities m, e, e' and γ.

Now, two of the Moon's coordinates, viz. the longitude and latitude, can be directly compared with observation, whereas

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the third coordinate, viz. the radius vector, can only be indirectly inferred from observation through the parallax, to the sine of which it is inversely proportional.

Hence the accuracy of the theoretical values of the longitude and latitude can be much more severely tested by observation than that of the radius vector.

Delaunay has, on account of this circumstance, found the analytical expressions for the longitude and latitude with a much greater degree of accuracy than that for the reciprocal of the radius vector.

In the two former coordinates he has taken into account generally the terms of the 7th order, and in cases where the convergence of the series is found to be slow, he has included terms of the 8th and 9th orders. In the reciprocal of the radius vector, however, he has confined his attention to terms of the 5th order. Consequently, while the coefficients of the inequalities in longitude and latitude as found by him are generally only a small fraction of a second in error, the inequalities in the reciprocal of the radius vector are not found with sufficient precision to give even the parallax itself with all the accuracy which is desirable.

The coefficients of the inequalities of the parallax given by me in vol. xiii. of the *Monthly Notices*, p. 263, are considerably more accurate than those of Delaunay.

In the paper just referred to, I have given the coefficients to hundredths of a second only, and, as I have there stated, terms with coefficients less than 0″·05 have been omitted except when they can be included in the same table with larger terms.

It may be worth while to give here a more complete view of the values of the coefficients of parallax which I obtained in 1853. These results are exhibited to thousandths of a second, as the calculation gave them, although the figures in the last place of decimals are not to be depended upon.

I add, for the sake of comparison, Delaunay's coefficients of the corresponding terms as given in the *Connaissance des Temps* for 1869, and also the coefficients of Hansen's theory as transformed by Professor Newcomb. The several arguments are expressed in Delaunay's notation.

Table of Comparative Values of the Coefficients of $\frac{\textit{sin. Parallax}}{\textit{sin. } 1''}$.

Arg.	Delaunay.	Adams.	Hansen transformed by Newcomb.
0	3422″·7	3422″·324	3422″·09
l'	−0·4273	−0·400	−0·393
l	+186·5870	186·513	186·483
$2l$	10·1984	10·170	10·161
$3l$	0·6314	0·628	0·620

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Arg.	Delaunay.	Adams.	Hansen transformed by Newcomb.
$4l$	″ ·0414	″ ·041	″ ·040
$l-l'$	1·0523	1·157	1·144
$l+l'$	−0·9118	−0·948	−0·961
$2l-l'$	0·1030	0·123	0·149
$2l+l'$	−0·0917	−0·100	−0·122
$2F-l$	−0·7079	−0·710	−0·709
$2D$	28·1788	28·232	28·225
$2D-l'$	1·8764	1·915	1·920
$2D+l'$	−0·3276	−0·306	−0·301
$2D-2l'$	0·0760	0·089	0·092
$2D+l$	3·0636	3·090	3·084
$2D+l-l'$	0·1967	0·222	0·229
$2D+l+l'$	−0·0401	−0·047	−0·049
$2D-l$	34·1662	34·304	34·309
$2D-l-l'$	1·4523	1·449	1·447
$2D-l-2l'$	0·0454	0·050	0·049
$2D-l+l'$	−0·3789	−0·231	−0·227
$2D-2l$	0·2707	0·281	0·283
$2D-2l$	−0·2770	−0·307	−0·302
$2D-3l$	−0·1012	−0·116	−0·121
$2D-2F$	−0·1092	−0·106	−0·105
$2D-2F+l$	−0·0501	−0·048	−0·048
$2D-2F-l$	−0·0816	−0·086	−0·083
$4D$	0·1960	0·260	0·261
$4D-l$	0·4991	0·600	0·599
$4D-2l$	0·3104	0·372	0·372
$4D-l-l'$	0·0297	0·063	0·069
D	−0·9378	−0·949	−0·953
$D+l'$	0·1507	0·145	0·146
$D+l$	−0·0971	−0·106	−0·106
$3D$	0·0158	0·005	0·003
$3D-l$	−0·0199	−0·036	−0·037
$3D+l$	0·0025	0·002	
$D-l$	0·0076	0·014	+0·011
$2D-2l-l'$	−0·0127	−0·015	−0·019
$4D+l$	0·0185	0·032	0·043
$4D-2l-l'$	0·0159	0·030	0·032
$4D-l'$	0·0110	0·034	0·035

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In the above many very small coefficients have been omitted.

As stated in my paper in the appendix to the *Nautical Almanac* for 1856, or in the *Monthly Notices*, vol. xiii. p. 177, my coefficients of parallax were obtained by comparing the results of the theories of Damoiseau, Plana, and Pontécoulant, and tracing out the origin of the discordances in the cases where those results did not agree with each other. These coefficients were also compared with those which I obtained by a transformation of Hansen's preliminary results as given in a paper in vol. xvii. of the *Astronomische Nachrichten.*

In Pontécoulant's method the expression for the reciprocal of the radius vector is first found, and then the expression for the longitude is derived from it. Hence the analytical values of the coefficients of parallax, given by Pontécoulant, vol. iv. pp. 149–152, 281, 282, 336, 337, are at least as accurate as the values of his coefficients of longitude.

In his final expression, however, in pp. 568–572, in which the several terms of the reciprocal of the radius vector are collected together, he neglects all terms of orders higher than the 5th, and the same omission takes place in the conversion of his coefficients of parallax into numbers.

Accordingly these numerical values, which are calculated in pp. 599–601, and collected together in p. 635, nearly coincide with the values of Delaunay, but are on the whole still less accurate.

It is greatly to be desired that some intrepid and competent calculator would undertake to make the numerous substitutions which would be required in order to find, by Delaunay's method, the expression for the reciprocal of the radius vector to the same order of accuracy as that which Delaunay has already attained in the case of the corresponding expressions for the longitude and latitude. The work would be one of simple substitution, not requiring the solution of any new equations, and consequently its only difficulty would consist in its great length.

The fact that Delaunay's determination of the value of the reciprocal of the radius vector is a comparatively rough one, affords a ready explanation of a difficulty which Sir George Airy has recently met with in his Numerical Lunar Theory.

The first operation required in this method is the substitution in the differential equations of motion of the numerical values of the Moon's coordinates as obtained in Delaunay's theory. If the theory were exact, the result of the substitution in each equation would be identically zero, so that the coefficient of each separate term in the result of the substitution would vanish. In consequence of errors in the coefficients obtained by Delaunay, however, this mutual destruction of terms will not take place, and the result of the substitution will consist of a number of terms the coefficients of which will depend on the errors of the assumed coefficients.

If, as is actually the case, these latter errors be so small that

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their squares and products may be neglected, each of the residual coefficients may be represented by a linear function of the errors of the assumed coefficients, and the formation of the corresponding linear equations constitutes the second operation in Sir George Airy's method. The solution of these linear equations by successive approximations will finally give the corrections which must be applied to Delaunay's coefficients in order to satisfy the differential equations.

Now, since the proportionate errors of Delaunay's coefficients of parallax are considerable, and much greater than the errors affecting his coefficients of longitude and latitude, it will be readily understood that the result of the substitutions will be to leave considerable residual coefficients in the two equations which relate to motion parallel to the ecliptic, and much smaller residual coefficients in the third equation which relates to motion normal to the ecliptic, since in this last equation every error in the coefficients of the radius vector or of its reciprocal will be multiplied by the sine of the inclination of the Moon's orbit. This result, which might thus have been anticipated, is exactly what Sir George Airy has found to take place, according to a memorandum which he has recently addressed to the Board of Visitors of the Royal Observatory.

Since the errors affecting Delaunay's coefficients of parallax are comparatively large, it will be necessary to determine the factors by which these errors are multiplied in the equations of condition with a much greater degree of accuracy than is required in the case of the factors by which the errors of the coefficients of longitude and latitude are multiplied in the same equations. Otherwise, it will not be possible to deduce these last-mentioned errors from the equations with the requisite degree of precision. It will be necessary to take special precautions in order to determine with accuracy the corrections of the assumed coefficients in the inequalities of longitude which have long periods.

On the Change in the Adopted Length of the "Tabular Mean Solar Day," which takes place with every Change in the Adopted Value of the Sun's Mean Sidereal Motion. By E. J. Stone, M.A., F.R.S.

The view which I have taken that the so-called "mean solar day" of any of our tables is not necessarily the true "mean solar day," and that the "mean solar day" in use before 1864, for the calculation of our *Nautical Almanac*, is most certainly not the same interval of absolute time as that now adopted, and called a "mean solar day," is without doubt a startling assertion, and one likely to be sharply criticised; but I did not make it without a most careful consideration of the subject, and I feel perfectly certain of its truth. After all, the statement is not more startling than the facts for which it accounts. The meri-